Appendices

Appendix 1: Solving problem

A1-§1: The calculation here is based on the nozzle equations derived for ideal gas and lossless flow. This is a methodical problem, where the calculation of the required quantities can be done in the order as given in the entered parameters, i.e. first we decide whether critical flow occurs, then we calculate the outlet velocity and finally we calculate the mass flow through the nozzle.

Entered parameters of the problem are:

V_i	p_i	t_i	p_e	A_e	c_p	r	κ
250	1	350	0,25	15	1,01	287	1,4

V [m·s^{-1}]; p [MPa]; t [°C]; A [cm²]; c_p [kJ·kg^{-1}·K^{-1}]; r [J·kg^{-1}·K^{-1}]; κ [1]

A1-§2: To determine if the flow in the nozzle is critical, compare the stagnation nozzle pressure ratio ε_s with the critical pressure ratio for dry air ε^*_s. The nozzle static pressure ratio ε can also be used. If the static pressure ratio ε is less than the pressure ratio ε^*_s, it is certain that critical flow will occur.

The critical pressure for dry air can be read from Table 4:

ε^*_s
0,5283

[1]

A1-§3: The critical pressure ratio from the static pressures ε can be calculated using Equation 5 by substituting p_i into the pressure numerator instead of p_{is}:

ε
0,25

[1]

A1-§4: If $\varepsilon<\varepsilon^*_s$ is valid, it means that critical conditions will occur, therefore the quantities at the nozzle outlet will be indexed* to show that this is a critical condition at this point.

A1-§5: The calculation of the outlet velocity V_e can be based on Equation 1(b). In this case the total absolute temperature T_{is}, must be determined, the pressure ratio in the nozzle throat will be critical.

We calculate the total temperature t_{is} using the definition equation of the total enthalpy h_s (see also Figure 1) equation to calculate the enthalpy as a function of heat capacity temperature at constant pressure [Škorpík, 2024]:

$$h_{is}=h_i+\frac{V_i^2}{2},$$

$$C_p\cdot t_{is}=C_p\cdot t_i+\frac{V_i^2}{2},$$

$$t_{is}=t_i+\frac{V_i^2}{C_p\cdot 2}.$$

t_{is}	T_{is}	V^*_e
380,94	654,09	467,97

t [°C]; T [K]; V [m·s^{-1}]

A1-§6: If the critical state is reached in the nozzle, then Equation 7 can be used for the mass flow through the nozzle.

The outlet coefficient χ_{max} can be read from Table 4:

χ_{max}
0,6847

[1]

The stagnation pressure at the nozzle inlet p_{is} can be calculated from the isentropy equation and the ideal gas equation of state [Škorpík, 2024]:

$$p_{is} \cdot v_{is}^{\kappa} = p_i \cdot v_i^{\kappa},\ v = \frac{r \cdot T}{p} \Rightarrow p_{is} = p_i \left(\frac{T_{is}}{T_i} \right)^{\frac{\kappa}{1-\kappa}}.$$

The equation of state of an ideal gas can also be used to calculate the specific volume v_{is}:

p_{is}	v_{is}	m^*
1,1848	158,44	2,8086

p [MPa]; v [dm^3·kg^{-1}]; m [kg·s^{-1}]

Appendix 2: Solving problem

A2-§1: The calculation here is based on the nozzle equations derived for ideal gas and lossless flow. Equation 13 is used to calculate the Laval nozzle dimensions.

Entered parameter of the problem is:

α
10

α [°]

A2-§2: To calculate the radius of the nozzle, or its length, it is necessary to know the flow flow area at the inlet (see Problem 1) and at the outlet, which can be calculated from the mass flow rate through the nozzle, the gas parameters (Problem 1) and the continuity equation for the nozzle (Equation 3). The state variables are calculated according to the equations for an ideal gas (isentropy equation, equation of state) [Škorpík, 2024], so for the specific volume v_e we can write:

$$p_{is} \cdot v_{is}^{\kappa} = p_e \cdot v_e^{\kappa} \Rightarrow v_e = v_{is} \left(\frac{p_{is}}{p_e} \right)^{\frac{1}{\kappa}}.$$

A^*	$\dot{m}$	κ	p_{is}	v_{is}	p_e	ε_s	r^*	V_e
15	2,8086	1,4	1,1848	158,44	0,25	0,2110	2,1851	686,73
v_e	A_e	r_e	r_r	t	r_t	l		
481,40	19,689	2,5034	0,8347	0,0727	2,1883	3,6747		

A [cm²]; m [kg·s^{-1}]; κ [1]; p [MPa]; v [dm^3·kg^{-1}]; ε [1]; r, t, l [cm]; V [m·s^{-1}]

A2-§3: The Mach number and the speed of sound[3.] can be calculated using the following equations:

$$a_e = \sqrt{\kappa \cdot r \cdot T_e};\ M_e = \frac{V_e}{a_e}.$$

The product $r \cdot T_e$ can be substituted by the gas equation of state $p_e \cdot v_e$:

a_e	M_e
410,48	1,6730

a [m·s^{-1}]; M [1]