# Appendices

**Appendix 1:**

**A1.§1:**
Solving problem

The calculation here is based on the nozzle equations derived for ideal gas and lossless flow. This is a methodical problem where we can proceed to calculate the required quantities in the order as they are given in the assignment, i.e. first we decide whether the critical flow occurs, then we calculate the outlet velocity and finally we calculate the mass flow through the nozzle.

Procedure for solving Problem 1

| | | | | | |
|---|---|---|---|---|---|
| **§1** entry: | $V_i$; $p_i$; $t_i$; $p_e$; $A_e$; $C_p$; $r$; $\kappa$ | **§5** calculation: | $T_{is}$; $V^*_e$ |
| **§2** read off: | $\varepsilon^*_s$ | **§6** read off: | $\chi_{max}$ |
| **§3** calculation: | $\varepsilon$ | **§7** calculation: | $p_{is}$; $v_{is}$; $m^*$ |
| **§4** compare: | $\varepsilon^*_s$ vs. $\varepsilon$ | | |

Entered parameters of the problem are:

| $V_i$ | $p_i$ | $t_i$ | $p_e$ | $A_e$ | $C_p$ | $r$ | $\kappa$ |
|---|---|---|---|---|---|---|---|
| 250 | 1 | 350 | 0,25 | 15 | 1,01 | 287 | 1,4 |

$V$ [m·s$^{-1}$]; $p$ [MPa]; $t$ [°C]; $A$ [cm²]; $C_p$ [kJ·kg$^{-1}$·K$^{-1}$]; $r$ [J·kg$^{-1}$·K$^{-1}$]; $\kappa$ [1]

**A1.§2:**

We can determine if the flow in the nozzle is critical by comparing the total nozzle pressure ratio $\varepsilon_s$ with the critical pressure ratio for dry air $\varepsilon^*_s$. It can also be easier to use the nozzle static pressure ratio $\varepsilon$, which can be easily determined from the specification, if this is less than the pressure ratio $\varepsilon^*_s$, then it is certain that critical flow will occur because $\varepsilon_s > \varepsilon$.

The critical pressure for dry air can be read from Table 4:

| $\varepsilon^*_s$ |
|---|
| 0,5283 |

[1]

**A1.§3:**

The critical pressure ratio from the static pressures $\varepsilon$ can be calculated using Equation 5 by substituting $p_i$ into the pressure numerator instead of $p_{is}$:

| $\varepsilon$ |
|---|
| 0,25 |

[1]

**A1.§4:**

If $\varepsilon<\varepsilon^*_s$ is valid, it means that critical conditions will occur, therefore the quantities at the nozzle outlet will be indexed$^*$ to show that this is a critical condition at this point.

**A1.§5:**

The calculation of the outlet velocity $V_e$ can be based on Equation 1(b). In this case the total absolute temperature $T_{is}$, must be determined, the pressure ratio in the nozzle throat will be critical.

We calculate the total temperature $t_{is}$ using the definition equation of the total enthalpy $h_s$ (see also Figure 1) equation to calculate the enthalpy as a function of heat capacity temperature at constant pressure [Škorpík, 2019]:

$$h_{is}=h_i+\frac{V_i^2}{2},$$

$$C_p\cdot t_{is}=C_p\cdot t_i+\frac{V_i^2}{2},$$

$$t_{is}=t_i+\frac{V_i^2}{C_p\cdot 2}.$$

| $t_{is}$ | $T_{is}$ | $V^*_e$ |
|---|---|---|
| 380,94 | 654,09 | 467,97 |

$t$ [°C]; $T$ [K]; $V$ [m·s$^{-1}$]

**A1.§6:**

If the critical state is reached in the nozzle, then Equation 7 can be used for the mass flow through the nozzle.

The outlet coefficient $\chi_{max}$ can be read again from Table 4:

| $\chi_{max}$ |
|---|
| 0,6847 |

[1]

The stagnation pressure at the nozzle inlet $p_{is}$ can be calculated from the isentropy equation and the ideal gas equation of state [Škorpík, 2019]:

$$p_{is}\cdot v_{is}^{\kappa}=p_i\cdot v_i^{\kappa},\ v=\frac{r\cdot T}{p}\Rightarrow p_{is}=p_i\left(\frac{T_{is}}{T_i}\right)^{\frac{\kappa}{1-\kappa}}.$$

The equation of state of an ideal gas can also be used to calculate the specific volume $v_{is}$:

| $p_{is}$ | $v_{is}$ | $m^*$ |
|---|---|---|
| 1,1848 | 158,44 | 2,8086 |

$p$ [MPa]; $v$ [dm$^3$·kg$^{-1}$]; $m$ [kg·s$^{-1}$]

**Appendix 2:**

**A2.§1:** Solving problem

The calculation here is based on the nozzle equations derived for ideal gas and lossless flow. Equation 13 is used to calculate the CD nozzle dimensions.

Procedure for solving Problem 2

| | | | | | |
|---|---|---|---|---|---|
| **§1** | entry: | $\alpha$ | **§3** | calculation: | $a_e$; $M_e$ |
| **§2** | calculation: | $\varepsilon_s$; $r^*$; $V_e$; $v_e$; $A_e$; $r_e$; $r_r$; $t$; $r_t$; $l$ | | | |

Entered parameter of the problem is: